Algorithms

Canvas: Groups & Notes

Background:

logarithms,

induction, etc.

Last bits from Canvas/Syllabus:

- HWO groups: <u>you</u> sign up
- Perusall: Sorry for difficulties! Everyone in now?

<u>Note</u>: Given all the potential for issues (websites, COVID, internet dependence), please know:

- I'll try to stay flexible about one-time problems
- I'll drop lowest 1-2 of various assignments

Communication:

- Discord: optional, but a nice idea! Please treat it as a class gathering area, & I'll try to answer questions

- Office hours: thoughts? Zoom, discord, etc. ?

For this week:

Thursday: 10-11am → discord
Friday: after class → zoom
Tuesday: 2pm-3pm → discord or zoom?

Please stop in!!

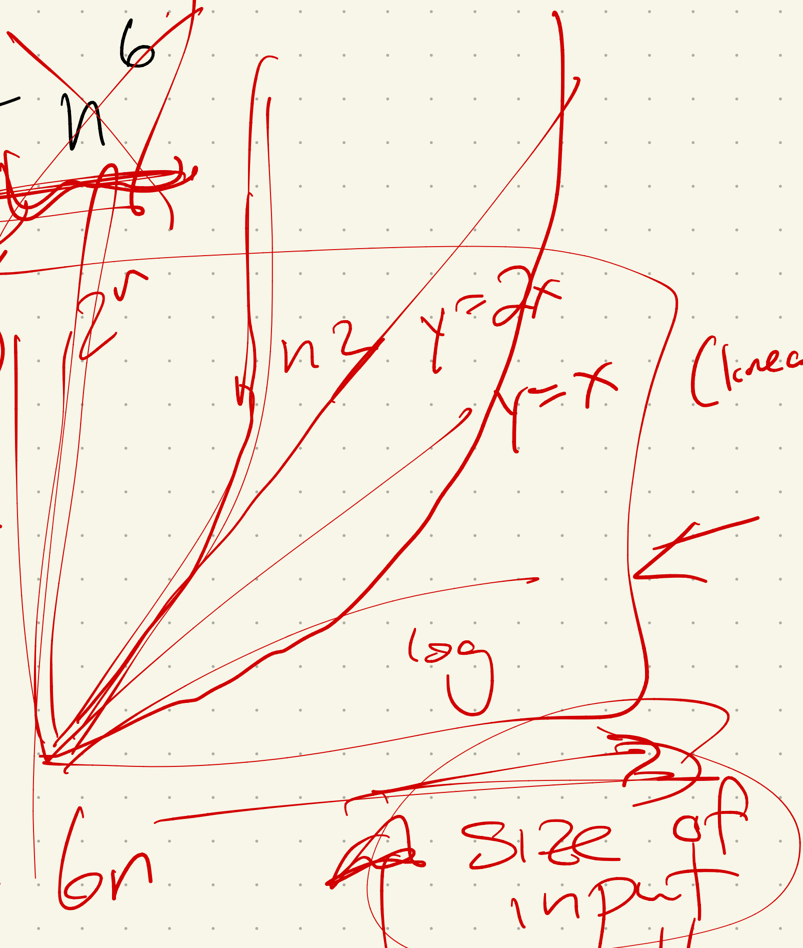
Some background reminders

① Big-O: ignore constants!

Also, smaller terms "disappear"

Ex: $6\cdot 2^n + \cancel{n^6}$

$n^6 < c\cdot(6\cdot 2^n)$



Big O: running of alg

$6\cdot 2^n < c\cdot 2^n$

$c=7$

$n^2 - 6n$

remove all those "noisy" factors

$O(2^n)$

(see Ch. in Data Structures)

② logarithms: useful identities!

Find it in your discrete math reference, ie →

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^y) = y\log_b(x)$$

$$\log_b(\sqrt[y]{x}) = \frac{\log_b(x)}{y}$$

Note:
$2^x \cdot 2^y = 2^{x+y}$
$(2^x)^y = 2^{xy}$

Why?

$$\log_5 7 = \frac{\log_3 7}{\log_3 5}$$

$x=3$

$$\boxed{\log_a b} = \frac{\log_x b}{\log_x a}$$ with any base x

Ex: Is $\log_{10} n$ big-O of $\log_2 n$?

$$\log_{10} n = \frac{\log_2 n}{\log_2 10}$$

$$= \underbrace{\left(\frac{1}{\log_2 10}\right)}_{=c} \log_2 n = \boxed{O(\lg n)}$$

lg: base 2

Another: $\log_a(a^x) = x$

by definition

identity: $\boxed{\log_a(b^x) = x\log_a b}$

$\log_2(2^n) = n\underbrace{\log_2 2}_{1} = n$

Note: $(\log n)^2 \neq 2\log n$

Ex: Give big-O of

$\log(n^2)$ $\log n^2$

$= \underbrace{2}_{\text{constant}}\log n$

$= O(\log n)$

③ Summations:

again, your discrete math book has a table.
Find it.

Ex:

Helpful Summation Identities

$\sum_{i=1}^{n} c = nc$	for every c that does not depend on i	(1)
$\boxed{\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$	Sum of the first n natural numbers	(2)
$\sum_{i=1}^{n} 2i - 1 = n^2$	Sum of first n odd natural numbers	(3)
$\sum_{i=0}^{n} 2i = n(n+1)$	Sum of first n even natural numbers	(4)
$\sum_{i=1}^{n} \log i = \log n!$	Sum of logs	(5)
$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$	Sum of the first squares	(6)
$\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$	Nichomacus' Theorem	(7)
$\sum_{i=0}^{n-1} a^i = \frac{1-a^n}{1-a}$	Sum of geometric progression	(8)
$\sum_{i=0}^{n-1} \frac{1}{2^i} = 2 - \frac{1}{2^{n-1}}$	Special case for $n = 2$	(9)
$\sum_{i=0}^{n-1} ia^i = \frac{a - na^n + (n-1)a^{n+1}}{(1-a)^2}$		(10)
$\sum_{i=0}^{n-1} (b+id)a^i = b\sum_{i=0}^{n-1} a^i + d\sum_{i=0}^{n-1} ia^i$		(11)
		(12)

Notation: $\sum_{i=0}^{n} \underbrace{f(i)}_{i \text{ or } \log i}$

linear

$$\boxed{\sum_{i=1}^{n} \left[\sum_{j=1}^{i} 1 \right]}$$

nested for loops

$$\sum_{j=0}^{i-1} 1 = \underbrace{1+1+1\cdots+1}_{i}$$

$$\sum_{i=0}^{n-1}\underbrace{\left(\sum_{j=0}^{i-1} 1\right)}_{i}$$

for $i \leftarrow 1$ to n
  for $j = 1$ to i
    $A[i][j] = 0$

$$= \sum_{i=0}^{n-1} i = 0 + 1 + 2 + 3 + \cdots + \overset{n-2}{(n-1)}$$

$$= \cancel{O(n^2)}$$

$$\approx \frac{n}{2}(n-1) = O(n^2)$$

④ Induction:

There is a template.

Base case: n = some small value

Ind hypothesis: Assume true for k-1 or less

Ind. step: Assuming IH, show it holds for the next value, k

1 = small → 2 → 3 → . . . .

rec function:

Base case:

if n > base:

divide + conquer

Use it! Love it!

3) Pseudocode conventions:

Variable assignment:

$x \leftarrow 5$

Boolean comparison:

if $(x = 5)$

in C:

~~if $(x == 5)$~~

Arrays: $A[0..n-1]$

- each $A[i]$: item in list

Loops: for $v \leftarrow A[0..n-1]$

A[0] then A[1] ...

iterator-like

Again, takes practice! Read intro chapter + then give HW0 a try.

6 Any questions so far?